



**AIAA-2000-0474**

**Accuracy Study of the Space-Time CE/SE  
Method for Computational Aeroacoustics  
Problems Involving Shock Waves**

Xiao-Yen Wang

Taitech Inc.

Cleveland, OH

and

Sin-Chung Chang and Philip C.E. Jorgenson

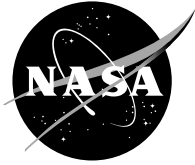
NASA Glenn Research Center

Cleveland, OH

**38th Aerospace Sciences**

**Meeting & Exhibit**

January 10-13, 2000 / Reno, NV



# Accuracy Study of the Space-Time CE/SE Method for Computational Aeroacoustics Problems Involving Shock Waves

Xiao-Yen Wang  
Taitech Inc., Cleveland, Ohio

Sin-Chung Chang and Philip C.E. Jorgenson  
Glenn Research Center, Cleveland, Ohio

## The NASA STI Program Office . . . in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the Lead Center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- **TECHNICAL PUBLICATION.** Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA's counterpart of peer-reviewed formal professional papers but has less stringent limitations on manuscript length and extent of graphic presentations.
- **TECHNICAL MEMORANDUM.** Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.
- **CONTRACTOR REPORT.** Scientific and technical findings by NASA-sponsored contractors and grantees.

- **CONFERENCE PUBLICATION.** Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or cosponsored by NASA.
- **SPECIAL PUBLICATION.** Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.
- **TECHNICAL TRANSLATION.** English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized data bases, organizing and publishing research results . . . even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at <http://www.sti.nasa.gov>
- E-mail your question via the Internet to [help@sti.nasa.gov](mailto:help@sti.nasa.gov)
- Fax your question to the NASA Access Help Desk at (301) 621-0134
- Telephone the NASA Access Help Desk at (301) 621-0390
- Write to:  
NASA Access Help Desk  
NASA Center for Aerospace Information  
7121 Standard Drive  
Hanover, MD 21076



# Accuracy Study of the Space-Time CE/SE Method for Computational Aeroacoustics Problems Involving Shock Waves

Xiao-Yen Wang  
Glenn Research Center, Cleveland, Ohio

Sin-Chung Chang and Philip C.E. Jorgenson  
Glenn Research Center, Cleveland, Ohio

Prepared for the  
38th Aerospace Sciences Meeting and Exhibit  
sponsored by the American Institute of Aeronautics and Astronautics  
Reno, Nevada, January 10-13, 2000

National Aeronautics and  
Space Administration

Glenn Research Center

## Acknowledgments

This work was supported by NASA Glenn Research Center through Contract NAS3-97186. The first author would like to thank Dr. Mark H. Carpenter for providing the analytical solution of the sound-shock interaction problem.

Available from

NASA Center for Aerospace Information  
7121 Standard Drive  
Hanover, MD 21076  
Price Code: A03

National Technical Information Service  
5285 Port Royal Road  
Springfield, VA 22100  
Price Code: A03

ACCURACY STUDY OF THE SPACE-TIME CE/SE  
METHOD FOR COMPUTATIONAL AEROACOUSTICS  
PROBLEMS INVOLVING SHOCK WAVES

Xiao-Yen Wang\*

Taitech Inc., NASA Glenn Research Center, Cleveland, OH 44135

and

Sin-Chung Chang<sup>†</sup> and Philip C.E. Jorgenson<sup>‡</sup>

NASA Glenn Research Center, Cleveland, OH 44135

### Abstract

The space-time conservation element and solution element(CE/SE) method is used to study the sound-shock interaction problem. The order of accuracy of numerical schemes is investigated. The linear model problem governed by the 1-D scalar convection equation, sound-shock interaction problem governed by the 1-D Euler equations, and the 1-D shock-tube problem which involves moving shock waves and contact surfaces are solved to investigate the order of accuracy of numerical schemes. It is concluded that the accuracy of the CE/SE numerical scheme with designed 2nd-order accuracy becomes 1st order when a moving shock wave exists. However, the absolute error in the CE/SE solution downstream of the shock wave is on the same order as that obtained using a fourth-order accurate essentially non-oscillatory(ENO) scheme. No special techniques are used for either high-frequency low-amplitude waves or shock waves.

### 1. Introduction

In the computational aeroacoustic problem, the propagation of disturbances of small amplitude needs to be captured. Thus a stringent requirement is placed on a numerical algorithm. And further, when a shock wave and acoustic waves exist at the same time, it becomes more challenging for a numerical simulation. In the current field of CAA, the high-order accuracy finite-difference method is popularly used for the propagation of high-frequency low-amplitude waves because a traditional 2nd-order accurate scheme is not accurate enough for this kind of problem. When a shock wave exists, the numerical scheme also has to be able to accurately cap-

ture the shock wave without oscillations. Traditional high-order accurate shock-capturing methods are classified into linear and nonlinear methods. The ENO schemes belong to the latter class. The order of accuracy of an ENO scheme is investigated for a sound-shock interaction problem in [1]. A very interesting conclusion drawn in [1] is that the accuracy of the solution downstream of the shock wave drops to first order if the shock wave is not located at a mesh point. The designed accuracy downstream of the shock wave can be achieved by using subcell resolution in a numerical algorithm such that the shock wave is located exactly on a mesh point. However, such a strategy is not practical for general multi-dimensional problems, since the shock wave could be curved and moving in the entire domain. Therefore, the advantage of using high-order accurate methods in the study of unsteady flows with shocks is questionable.

The space-time CE/SE method is an innovative numerical method for solving conservation laws. It is different in both concept and methodology from the well-established traditional methods such as the finite difference, finite volume, finite element and spectral methods. It is designed from a physicist's perspective to overcome several key limitations of the traditional numerical methods.

Simplicity, generality and accuracy are pursued in the development of the CE/SE method. Its salient properties are summarized briefly as follows. First, the concepts of conservation element and solution element are introduced to enforce both local and global flux conservations in space and time instead of in space only. Second, all the dependent variables and their spatial derivatives are considered as individual unknowns to be solved for simultaneously at each grid point. Third, no approximation techniques other than Taylor's series expansion, no monotonicity constraints, and no characteristic-based techniques are used in the design of the scheme. A detailed description of this method and the accompanying analysis are set forth in [2-4].

\*Research Scientist, Member AIAA  
email: wangxy@aem.umn.edu

<sup>†</sup>Aerospace Engineer, Member AIAA

<sup>‡</sup>Aerospace Engineer, Member AIAA

A variety of numerical tests have been performed previously to illustrate the robustness of this method. For the CE/SE Euler solver, highly accurate numerical solutions have been obtained for various flow problems involving discontinuities, such as shock waves, contact surfaces and even their interactions [5,6]. Moreover, applications of the same Euler solver to computational aeroacoustics (CAA) problems reveal that the accuracy of the results is comparable to that of a 4th-order compact difference scheme even though the current solver is only 2nd-order accurate. Further, the non-reflecting boundary conditions can be implemented in a simple way without involving characteristic variables. The solver can be applied to subsonic, transonic, and supersonic flows in the same form without using the characteristic-based techniques. Results show that the present solver can handle both continuous and discontinuous flows very well [7–14].

In this paper, the test problems used in [1] are solved by using the CE/SE method. Problems governed by the linear convection equation and the Euler equations are studied. The order of accuracy of the CE/SE numerical schemes is investigated. Numerical solutions are compared with those obtained by using the ENO scheme.

## 2. Numerical Test Problems

### 2.1. A linear model problem.

Consider the scalar equation

$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0 \quad (1)$$

where the wave speed  $a$  is

$$a = \begin{cases} 2, & x \leq x_s, \\ 1, & x > x_s \end{cases} \quad (2)$$

where  $x_s = 0.5$  given in [1]. The initial conditions are described as

$$u(x, 0) = \begin{cases} 1/2, & x \leq x_s, \\ 1, & x > x_s \end{cases} \quad (3)$$

The domain is  $0 \leq x \leq 1$ . The inflow boundary condition is

$$u(0, t) = (1 + \varepsilon \sin \omega t)/2 \quad (4)$$

where  $\varepsilon = 0.001$  and  $\omega = 8\pi$ . The period of the acoustic wave  $T_\lambda$  is define as  $2\pi/\omega$ . The outflow boundary condition is imposed by using the analytical solution given in [1]. The 1D CE/SE  $a$ - $\epsilon$ - $\alpha$  scheme constructed for Eq. (1) is used here with

$\epsilon = 0.5$ ,  $\alpha = 1$ , and Courant number 0.8. The details of the scheme are given in [2] and won't be repeated here.

First, an even number cells with 64, 128, 256, 512, 1024 uniform mesh intervals is used. The computed perturbation  $u'(x, t) = u(x, t) - u(x, 0)$  at  $t = 10T_\lambda$  and its  $\log_{10}$  error for different cell numbers are shown in Fig. 1. It can be seen that the scheme is 2nd-order accurate in the entire domain except at the shock location. In Fig. 2, the same results are shown for an odd number cells with 65, 129, 257, 513 uniform mesh intervals. It is still 2nd-order accurate in the entire domain since the space-time CE/SE method uses a staggered mesh. There is a grid point located at  $x_s = 0.5$  no matter whether an even or an odd number cells is used. An ENO-4-3 scheme, which is fourth-order accurate in space and third-order accurate in time, is used in [1]. For this linear problem, the accuracy of the ENO-4-3 solutions remains 4th order for an even number cells because there is a mesh point at the shock location, while the accuracy drops to first-order downstream of the shock wave for an odd number cells because the shock wave is within a cell.

Second,  $x_s = 0.5$  is replaced by  $x_s = 0.5 + ds$  with  $ds = 0.00035$ , which results in no mesh points coinciding with the shock location even on the finest mesh used. The solution is shown in Fig. 3. The accuracy downstream of the shock wave drops to first or zeroth order. It was found that the dominant error is from the phase error due to  $ds$  which can not be resolved. From this test result, it can be concluded that the designed 2nd-order accuracy can not be achieved if a shock is within a cell for the CE/SE scheme applied to linear problems. In order to obtain more relevant conclusion about the performance of the CE/SE method, the physical problem governed by the Euler equations is considered next.

### 2.2. Shock-Sound Interaction

The shock-sound interaction problem described in [1] is governed by the 1-D Euler equations, whose conservative form is expressed as

$$\frac{\partial u_m}{\partial t} + \frac{\partial f_m}{\partial x} = 0, \quad m = 1, 2, 3 \quad (5)$$

in which

$$u_1 = \rho, \quad u_2 = \rho v, \quad u_3 = p/(\gamma - 1) + \rho v^2/2 \quad (6)$$

and

$$f_1 = u_2 \quad (7)$$

$$f_2 = (\gamma - 1)u_3 + (3 - \gamma)(u_2)^2/(2u_1) \quad (8)$$

$$f_3 = \gamma u_2 u_3 / u_1 - (1/2)(\gamma - 1)(u_2)^3 / (u_1)^2 \quad (9)$$

where  $\rho, v, p$ , and  $\gamma$  are the mass density, velocity, static pressure, and constant specific heat ratio, respectively.

The spatial domain is  $0 \leq x \leq 1$ . The main flow is from left to right. A shock wave is initially located at  $x = 0.5$ . The initial conditions of main flow variables at the left side(upstream) and right side(downstream) of the shock wave are described as follows:

$$(\rho, v, p)_L = (1.0, 2.0, 1.0/1.4) \quad x < 0.5 \quad (10)$$

$$(\rho, v, p)_R = (2.6666, 0.75, 4.5/1.4) \quad x > 0.5 \quad (11)$$

The acoustic disturbance is introduced at  $x = 0$ . The flow variables at the inlet are defined as

$$p(0, t) = p_L(1 + \varepsilon \sin \omega t) \quad (12)$$

$$\rho(0, t) = \rho_L[p(0, t)/p_L]^{1/\gamma} \quad (13)$$

$$v(0, t) = v_L + \frac{2}{\gamma - 1}(c(0, t) - c_L) \quad (14)$$

where  $\omega = 36\pi$ ,  $\varepsilon = 0.001$ , and  $c = \sqrt{\gamma p/\rho}$  being the local sound speed, thus  $c_L = \sqrt{\gamma p_L/\rho_L}$  is the sound speed upstream of the shock wave. At the outlet( $x = 1$ ), the non-reflecting boundary condition is imposed to let flow propagate out the computational domain.

In this problem, the shock wave moves around the initial location due to the interaction with the acoustic wave. Thus, the shock wave is not located at a mesh point. Therefore, the first-order accuracy was achieved downstream of the shock wave using the ENO-4-3 scheme, which is shown in Fig. 4. However, in this problem, the shock wave does not move outside the cell in which it is initially located. Thus, inclusion of subcell resolution in the ENO-4-3 scheme can be done within this cell to exactly resolve the shock location. The 4th order accuracy downstream of the shock wave was achieved, which is shown in Fig. 5. Subcell resolution can be achieved more easily for the linear model problem described in 2.1, since the shock wave does not move and remains at  $x = 0.5$ . However, if shock waves are moving across the cells as the solution evolves, the use of subcell resolution in a scheme would be impractical due to its cost and complexity.

This problem is solved here again using the 1D CE/SE Euler solver. The same initial conditions and the boundary condition at the inlet are used. At the outlet( $x = 1$ ), the non-reflecting boundary condition which is not based on the characteristic theory is imposed as

$$(u_m)_j^n = (u_m)_{j-1/2}^{n-1/2}, \quad (u_{mx})_j^n = 0 \quad (15)$$

The boundary condition at the outlet is not mentioned for ENO scheme in [1]. Two parameters  $\epsilon$  and  $\alpha$  for controlling numerical dissipations in the CE/SE Euler solver are set as 0.5 and 1, and the Courant number is 0.8 in the calculations. The computed acoustic wave solutions  $p'(x, t) = p(x, t) - p(x, 0)$  and  $\rho'(x, t) = \rho(x, t) - \rho(x, 0)$  at  $t = 30T_\lambda$  are shown in Fig. 6 along with the exact solutions. The  $\log_{10}$  errors of both  $p'(x, t)$  and  $\rho'(x, t)$  are shown in Fig. 7. It can be concluded that the accuracy of the CE/SE solution is first order in the entire domain except at the shock location. The absolute error of the CE/SE solution downstream of the shock wave is on the same order as that of the ENO-4-3 solution shown in Fig. 4 using the same 512 uniform mesh intervals.

### 2.3. 1-D Shock-Tube Problem

Finally, the 1-D shock-tube problem named Sod's problem [2] is solved here in an accuracy study using the same solver mentioned in 2.2. In this problem, the shock wave and contact surface propagate through the entire spatial domain during the time interval of computation. The numerically computed solutions of pressure and density at  $t = 0.24$  with  $CFL = 0.44$  are shown in Fig. 8 along with the exact solution. The  $\log_{10}$  errors of pressure and density are shown in Fig. 9, respectively. It can be seen that the solution is first-order accurate in the entire domain except at the shock wave and contact surface locations. No solution obtained by an ENO scheme is available in [1].

### 3. Conclusions

Three test problems have been solved to investigate the accuracy of the CE/SE method for unsteady compressible flows with shock waves. Generally speaking, first-order accuracy can be obtained for the CE/SE method without using any special techniques for either high-frequency low-amplitude waves or shock waves. The absolute error in the CE/SE solution downstream of the shock wave is on the same order as that obtained by using fourth-order accurate ENO scheme for sound-shock interaction problem. It can be concluded that CE/SE method can produce accurate solutions for aeroacoustic problems involving shock waves in a simple way.

### Acknowledgments

This work was supported by NASA Glenn Research Center through Contract NAS3-97186. The first author would like to thank Dr. Mark H. Carpenter for providing the analytical solution of the sound-shock interaction problem.



## References

- [1] J. Casper and M. Carpenter, "Computational Considerations for the Simulation of Shock-Induced Sound", *SIAM J. SCI. COMPUT.*, Vol 19, No. 3, pp. 813-828, May 1998.
- [2] S.C. Chang, "The Method of Space-Time Conservation Element and Solution Element – A New Approach for Solving the Navier-Stokes and Euler Equations", *J. Comput. Phys.*, **119**, pp. 295-324, (1995).
- [3] S.C. Chang, X.Y. Wang, and C.Y. Chow, "The Space-Time Conservation Element and Solution Element Method – A New High-Resolution and Genuinely Multidimensional Paradigm for Solving Conservation Laws", to appear in *J. Comput. Phys.*.
- [4] X.Y. Wang and S.C. Chang, "A 2D Non-splitting Unstructured-triangular-mesh Euler Solver based on the Method of Space-Time Conservation Element and Solution Element", Vol. 8, No. 2, pp. 326-340, 1999, *Computational Fluid Dynamics JOURNAL*.
- [5] S.C. Chang, C.Y. Loh and S.T. Yu, "Computational Aeroacoustics via a New Global Conservation Scheme", *Proceedings of the 15th International Conference on Numerical Methods in Fluid Dynamics*, P. Kutler, J. Flores and J.-J. Chattot, eds., June 24-28, 1996, Monterey, CA.
- [6] S.C. Chang, A. Himansu, C.Y. Loh, X.Y. Wang, S.T. Yu and P.C.E. Jorgenson, "Robust and Simple Non-Reflecting Boundary Conditions for the Space-Time Conservation Element and Solution Element Method", AIAA Paper 97-2077, presented at the 13th AIAA CFD Conference, June 29-July 2, 1997, Snowmass, CO.
- [7] C.Y. Loh, S.C. Chang, J.R. Scott and S.T. Yu, "Application of the Method of Space-Time Conservation Element and Solution Element to Aeroacoustics Problems", *A Collection of Technical Papers, Volume II*, pp. 713-718, 6th International Symposium on Computational Fluid Dynamics, September 4-8, 1995, Lake Tahoe, NV.
- [8] C.Y. Loh, S.C. Chang, J.R. Scott and S.T. Yu, "The Space-Time Conservation Element Method – A New Numerical Scheme for Computational Aeroacoustics", AIAA Paper 96-0276, presented at the 34th AIAA Aerospace Sciences Meeting, January 15-18, 1996, Reno, NV.
- [9] C.Y. Loh, S.C. Chang and J.R. Scott, "Computational Aeroacoustics via the Space-Time Conservation Element / Solution Element Method", AIAA Paper 96-1687, presented at the 2nd AIAA/CEAS Aeroacoustics Conference, May 6-8, 1996, State College, PA.
- [10] C.Y. Loh, L.S. Hultgren and S.C. Chang, "Computing Waves in Compressible Flow Using the Space-Time Conservation Element and Solution Element method," AIAA Paper 98-0369, presented at the 36th AIAA Aerospace Sciences Meeting, January 12-15, 1998, Reno, NV.
- [11] X.Y. Wang, C.Y. Chow and S.C. Chang, "Numerical Simulation of Gust Generated Aeroacoustics in a Cascade Using the Space-Time Conservation Element and Solution Element Method," AIAA Paper 98-0178, presented at the 36th AIAA Aerospace Sciences Meeting, January 12-15, 1998, Reno, NV.
- [12] X.Y. Wang, C.Y. Chow and S.C. Chang, "Non-reflecting Boundary Conditions Based on the Space-Time CE/SE Method for Free Shear Flows," AIAA paper 98-3020, presented at the 29th AIAA Fluid Dynamics Conference, Albuquerque, New Mexico, June 15-18, 1998.

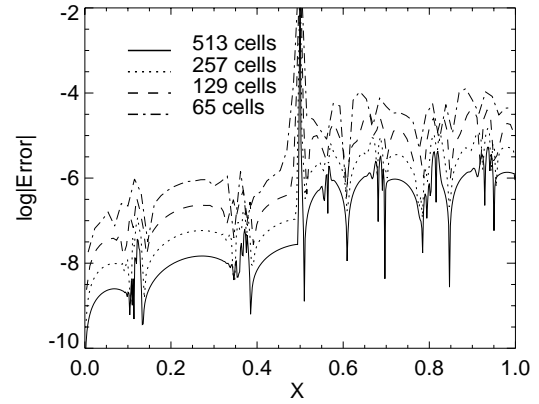
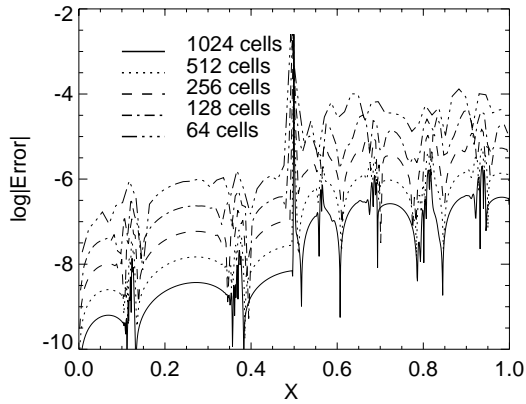
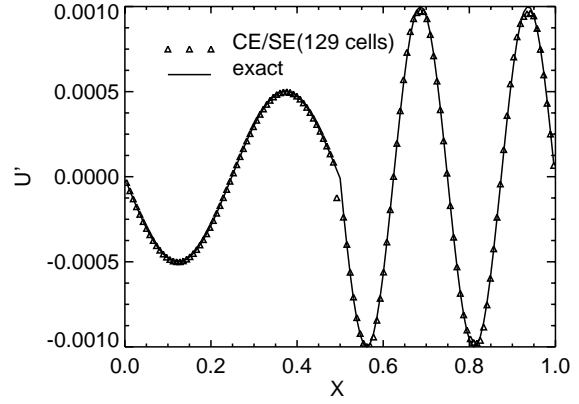
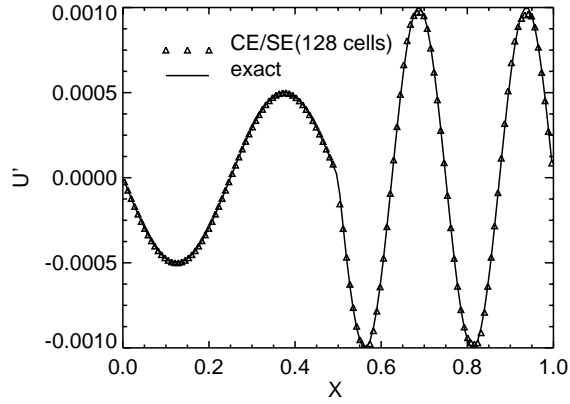


Figure 1: The CE/SE solution at  $t = 10T_\lambda$  (upper) and the pointwise error distribution using even numbered cells for a discontinuity located at  $x = 0.5$ .

Figure 2: The CE/SE solution at  $t = 10T_\lambda$  (upper) and the pointwise error distribution using odd numbered cells for a discontinuity located at  $x = 0.5$ .

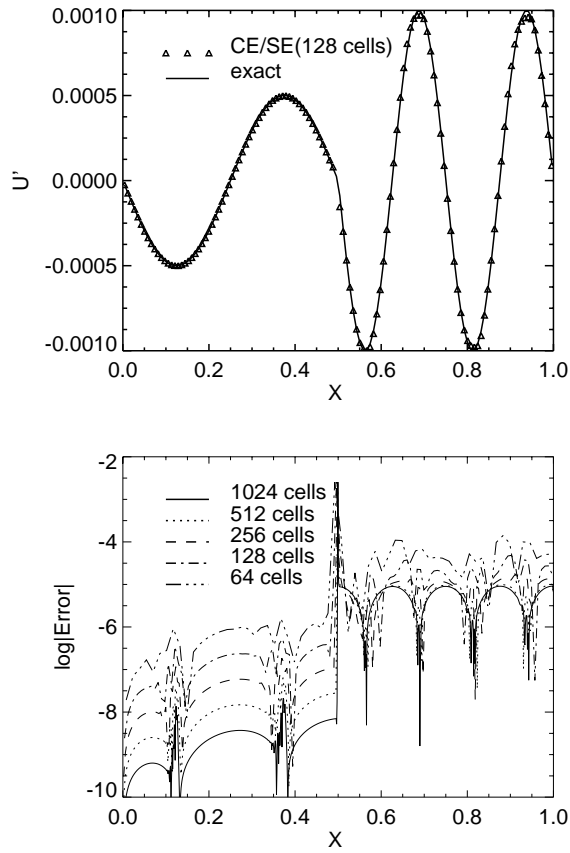


Figure 3: The CE/SE solution at  $t = 10T_\lambda$  (upper) and the pointwise error distribution using even numbered cells for a discontinuity located at  $x = 0.5 + 0.00035$ .

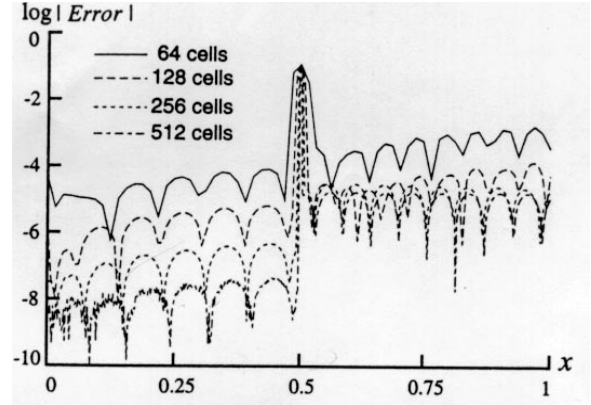


Figure 4: The ENO-4-3 pointwise error of pressure for the sound-shock interaction problem at  $t = 30T_\lambda$ .

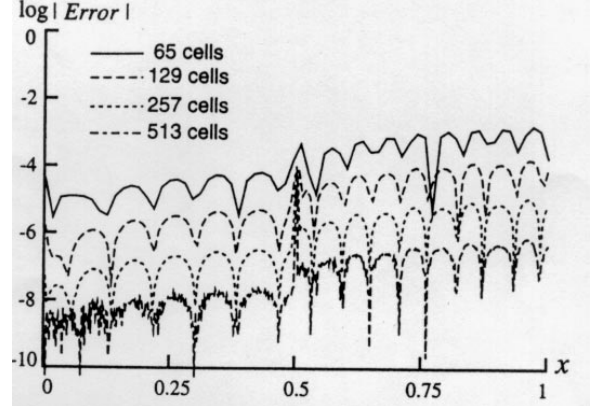


Figure 5: The ENO-4-3-SR pointwise error of pressure for the sound-shock interaction problem at  $t = 30T_\lambda$ .

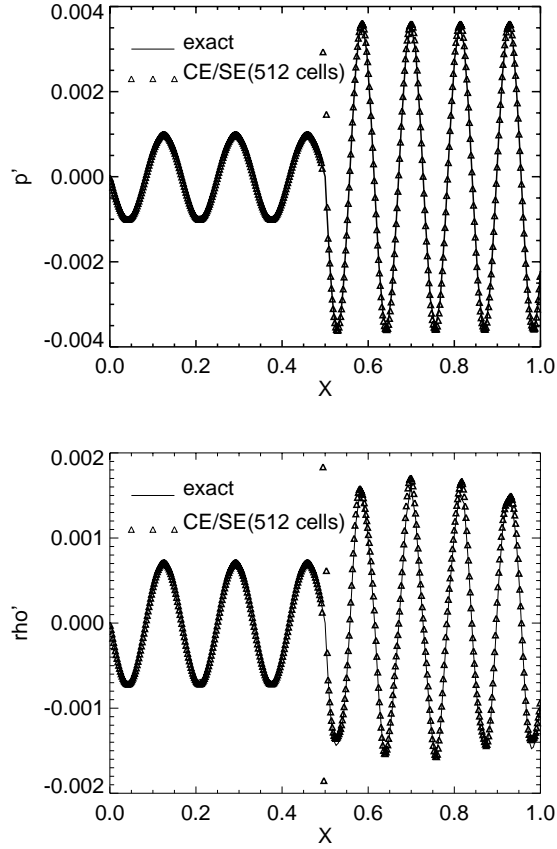


Figure 6: The CE/SE solutions of the sound-shock interaction problem at  $t = 30T_\lambda$ .

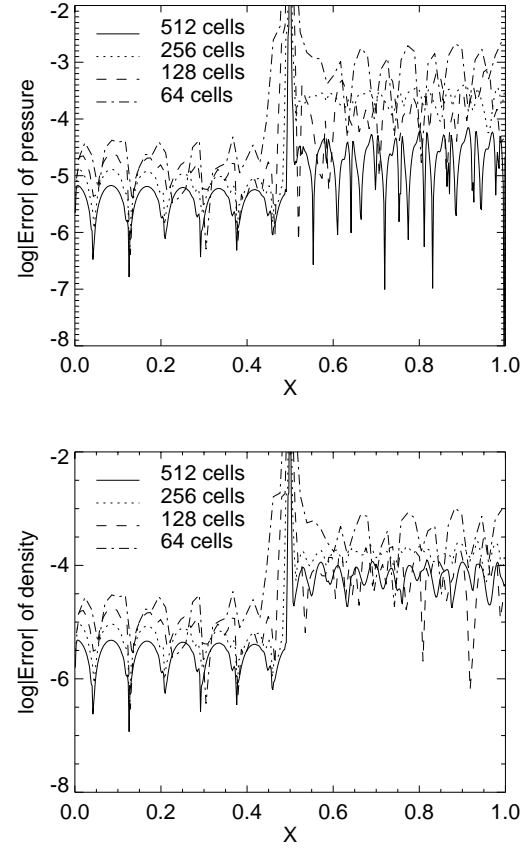


Figure 7: The CE/SE pointwise errors for the sound-shock interaction problem.

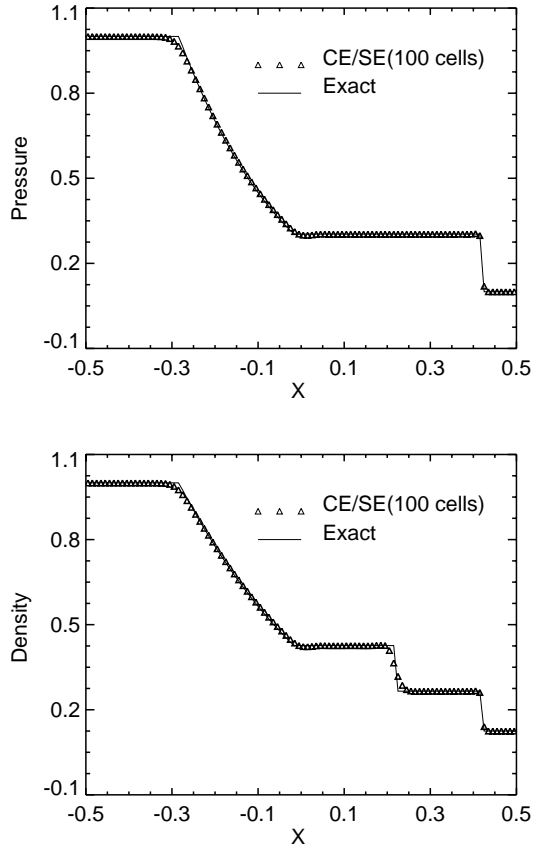


Figure 8: The CE/SE solutions of the shock-tube problem(Sod's problem) at  $t = 0.24$  with  $CFL = 0.44$ .

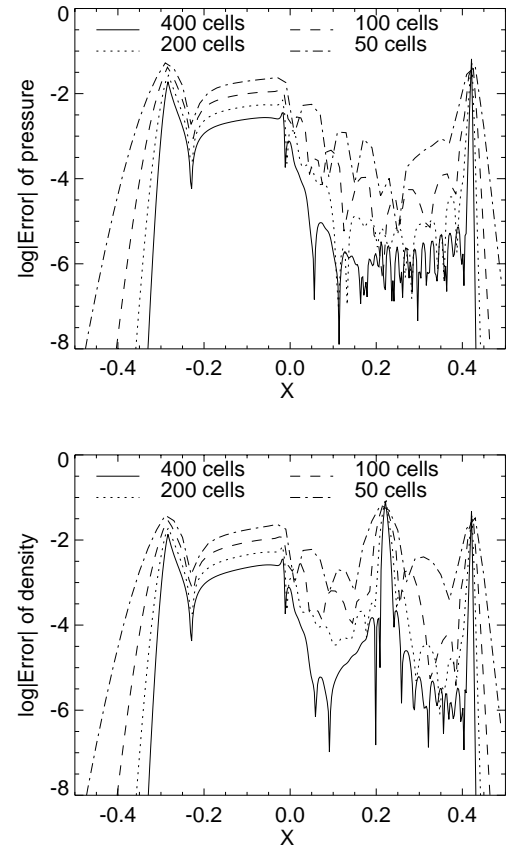


Figure 9: The CE/SE pointwise errors for the shock-tube problem with  $CFL = 0.44$ .

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE December 1999		3. REPORT TYPE AND DATES COVERED Technical Memorandum
4. TITLE AND SUBTITLE Accuracy Study of the Space-Time CE/SE Method for Computational Aeroacoustics Problems Involving Shock Waves			5. FUNDING NUMBERS  WU-522-81-11-00	
6. AUTHOR(S)  Xiao-Yen Wang, Sin-Chung Chang, and Philip C.E. Jorgenson				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration John H. Glenn Research Center at Lewis Field Cleveland, Ohio 44135-3191			8. PERFORMING ORGANIZATION REPORT NUMBER  E-12030	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Washington, DC 20546-0001			10. SPONSORING/MONITORING AGENCY REPORT NUMBER  NASA TM-1999-209659 AIAA-2000-0474	
11. SUPPLEMENTARY NOTES Prepared for the 38th Aerospace Sciences Meeting and Exhibit sponsored by the American Institute of Aeronautics and Astronautics, Reno, Nevada, January 10-13, 2000. Xiao-Yen Wang, Taitech Inc., Cleveland, Ohio, Sin-Chung Chang and Philip C.E. Jorgenson, NASA Glenn Research Center. Responsible person, Philip C.E. Jorgenson, organization code 5880, (216) 433-5386.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Unclassified - Unlimited Subject Category: 64  This publication is available from the NASA Center for AeroSpace Information, (301) 621-0390.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  The space-time conservation element and solution element(CE/SE) method is used to study the sound-shock interaction problem. The order of accuracy of numerical schemes is investigated. The linear model problem governed by the 1-D scalar convection equation, sound-shock interaction problem governed by the 1-D Euler equations, and the 1-D shock-tube problem which involves moving shock waves and contact surfaces are solved to investigate the order of accuracy of numerical schemes. It is concluded that the accuracy of the CE/SE numerical scheme with designed 2nd-order accuracy becomes 1st order when a moving shock wave exists. However, the absolute error in the CE/SE solution downstream of the shock wave is on the same order as that obtained using a fourth-order accurate essentially nonoscillatory (ENO) scheme. No special techniques are used for either high-frequency low-amplitude waves or shock waves.				
14. SUBJECT TERMS  Space-time conservation element and solution element method, Space-time flux, Euler equations, Numerical accuracy, Shock tube			15. NUMBER OF PAGES 14	
			16. PRICE CODE A03	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	



